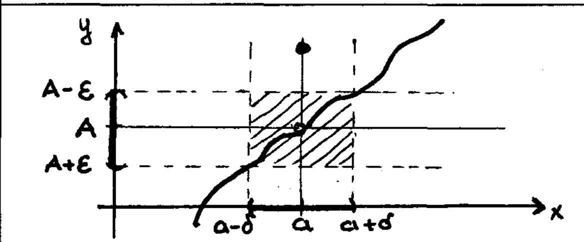
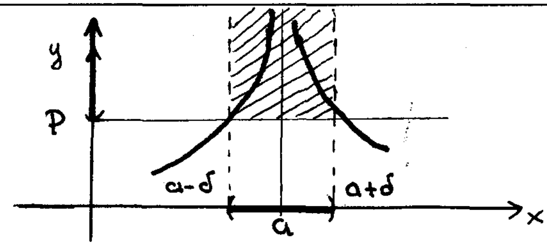
A $\lim\limits_{x\to a} f(x) = A$ definíciója

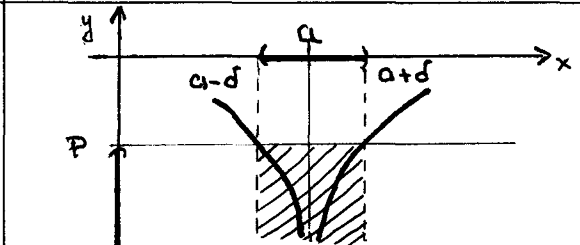
Legyen f valós-valós függvény és tegyük fel, hogy $a \in \mathcal{D}'_f$ (azaz a torlódási pontja $\mathcal{D}_f$-nek). Azt mondjuk, hogy az f függvénynek az a **pontban** $A \in \overline{\mathbb{R}}$ **a határértéke**, ha

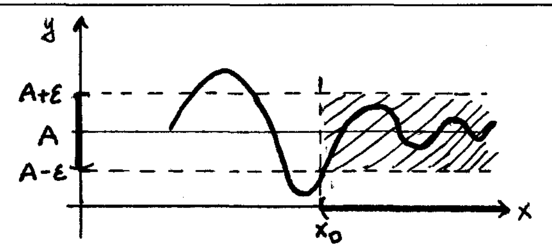
$$\forall\, \varepsilon > 0\text{-hoz} \quad \exists\, \delta > 0, \quad \text{hogy} \quad \forall\, x \in \big(k_\delta(a) \setminus \{a\}\big) \cap \mathcal{D}_f \quad \text{esetén} \quad f(x) \in k_\varepsilon(A).$$

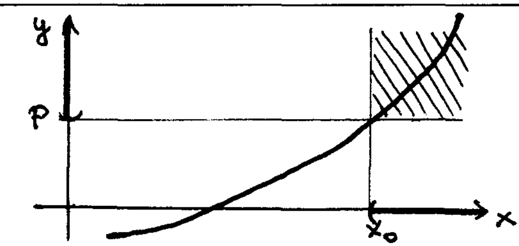
Attól függően, hogy a, illetve A valós szám vagy $\pm\infty$, ezt a definíciót egyenlőtlenségekkel a következőképpen fogalmazhatjuk meg:

<table>
<tr><th></th><th>$A \in \mathbb{R}$</th><th>$A = +\infty$</th><th>$A = -\infty$</th></tr>
<tr><td>$a \in \mathbb{R}$</td>
<td>

$\forall \varepsilon > 0\ \exists \delta > 0\ \ \forall x \in \mathcal{D}_f,\ 0 < |x-a| < \delta:\ \ |f(x) - A| < \varepsilon$</td>
<td>

$\forall P > 0\ \exists \delta > 0\ \ \forall x \in \mathcal{D}_f,\ 0 < |x-a| < \delta:\ \ f(x) > P$</td>
<td>

$\forall P < 0\ \exists \delta > 0\ \ \forall x \in \mathcal{D}_f,\ 0 < |x-a| < \delta:\ \ f(x) < P$</td></tr>
<tr><td>$a = +\infty$</td>
<td>

$\forall \varepsilon > 0\ \ \exists x_0 > 0\ \ \forall x \in \mathcal{D}_f,\ x > x_0:\ \ |f(x) - A| < \varepsilon$</td>
<td>

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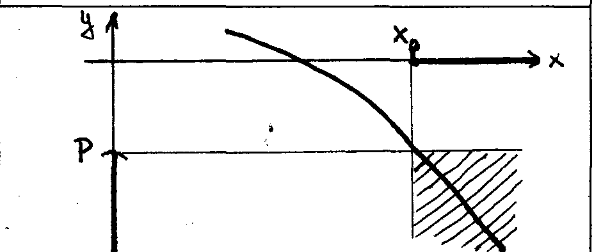
<td>

$\forall P < 0\ \ \exists x_0 > 0\ \ \forall x \in \mathcal{D}_f,\ \ x > x_0:\ \ f(x) < P$</td></tr>
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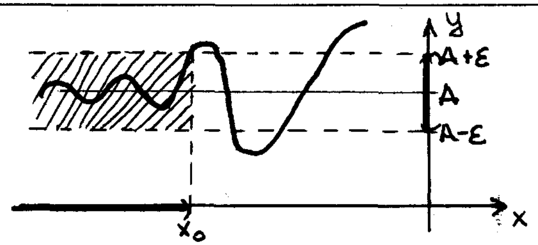
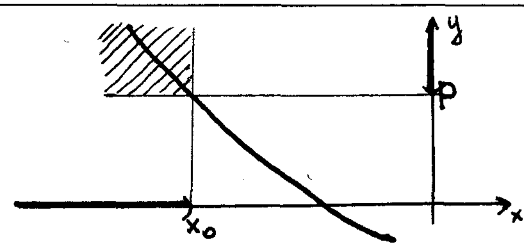
<td>

$\forall \varepsilon > 0\ \ \exists x_0 < 0\ \ \forall x \in \mathcal{D}_f,\ \ x < x_0:\ \ |f(x) - A| < \varepsilon$</td>
<td>

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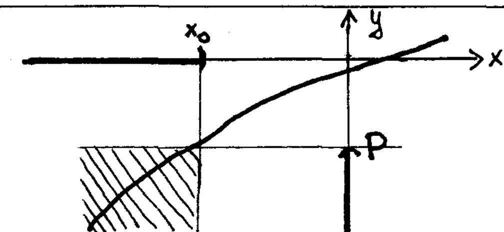
<td>

$\forall P < 0\ \ \exists x_0 < 0\ \ \forall x \in \mathcal{D}_f,\ x < x_0:\ \ f(x) < P$</td></tr>
</table>